Retardation Plates

A plate made of doubly refracting Crystal with its refracting surface cut ∥ to the optic axis and employed to introduce a phase diff. between O-ray & E-ray is called retarding plate.

If 'd' be the thickness of the plate, n_o & n_e are R.I. for O-ray & E-rays, then

Path diff. introduced by the plate is

$$(n_o \sim n_e)d.$$

The corresponding phase difference is

$$\delta = \frac{2\pi}{\lambda}(n_o \sim n_e)d$$

(A) Quarter Wave plate

If the thickness 'd' is such that the path diff. between O-Ray & E-ray is

$$(n_o \sim n_e)d = \lambda/4$$

$$\text{or, } d = \frac{\lambda}{4(n_o \sim n_e)}$$

Then the retarding plate is called Quarter wave plate or $\lambda/4$ plate.

(B) Half-Wave plate

$$\text{If, } (n_o \sim n_e)d = \lambda/2$$

$$\text{or, } d = \frac{\lambda}{2(n_o \sim n_e)}$$

The plate is called $\lambda/2$ plate.

Negative & Positive crystals (3)

(1) Negative crystals :- Crystals like calcite, in which O-ray travel with smaller velocity then E-ray, in a direction normal to the optic axis are called negative crystals.

For negative crystals

(i) $V_o < V_E$

(ii) $n_e < n_o$

$V_o \rightarrow$ vel. of ordinary ray
$V_e \rightarrow$ " " Extraord. ray
$n_e \rightarrow$ R.I. of E-ray
$n_o \rightarrow$ R.I " O-ray.

(2) Positive Crystal :- → Quartz

(i) $V_o > V_e$ → Vel. of O-Ray > vel. of E-ray.

(ii) $n_e > n_o$

Uniaxial Crystals

A doubly refracting crystal which have only one optic axis, is called Uniaxial crystal.

Dichroism

If an unpolarised beam of light is passed through a crystal like Tourmaline of proper thickness, one of the components among O-ray & E-ray is absorbed and other component is transmitted which is ~~The emergent ray is~~ linearly polarised. This phenomenon is called dichroism, and the crystal is called dichroic crystal.

Production of Circular & elliptically Polarised light — (mathematical Analysis.) (5)

"Let a plane polarised light be incident on a calcite plate cut with faces ∥ to optic axis. Let $\vec{E}$ vector makes an angle θ to the optic axis. On entering the crystal the amplitude of incident ray 'A' will be resolved into two components

(1) E – wave of ampl. $a = A\cos\theta$

(2) O – " " " $b = A\sin\theta$

Let the incident wave is represented by $E = A\cos(\omega t - KZ)$

Here, K → Propagation Vector

Z → Direction of Prop. of the Wave.

Then the component waves will be

$$x = a\cos(\omega t - KZ) \longrightarrow (1)$$

$$\& \quad y = b\cos(\omega t - KZ) \longrightarrow (2)$$

Now, E-ray & O-ray will travel with different vel. ($V_e > V_o$ in Calcite). So, a phase diff. δ will be introduced between the rays.

Hence, the emergent ray will be represented as —

$$x = a\cos(\omega t - KZ + \delta) \longrightarrow (3)$$

$$y = b\cos(\omega t - KZ) \longrightarrow (4)$$

From (3) $\quad \frac{x}{a} = \cos(\omega t - KZ + \delta) = \cos(\omega t - KZ)\cos\delta - \sin(\omega t - KZ)\sin\delta$

From (4) $\quad \frac{y}{b} = \cos(\omega t - KZ)$

$$\therefore \frac{x}{a} = \frac{y}{b}\cos\delta - \sqrt{1 - \frac{y^2}{b^2}}\,\sin\delta$$

or, $\left[\frac{x}{a} - \frac{y}{b}\cos\delta\right]^2 = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$

or, $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\delta + \frac{y^2}{b^2}\cos^2\delta$

$= \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$

or, $\frac{x^2}{a^2} - \frac{2xy\cos\delta}{ab} + \frac{y^2}{b^2}(\cos^2\delta + \sin^2\delta) = \sin^2\delta$

or, $\frac{x^2}{a^2} - \frac{2xy\cos\delta}{ab} + \frac{y^2}{b^2} = \sin^2\delta \rightarrow (5)$

Eqn (5) represents, in general, an ellipse confined in a rectangle of sides 2a and 2b. The nature of the resultant wave will depend upon phase diff. 'δ'.

(A) plane Polarised Light :-

if $\delta = 2m\pi$, $m = 0, 1, 2$ ----- etc

Then from (5)

For, $m = 0$, $\delta = 0$ } $\frac{x^2}{a^2} - \frac{2xy}{ab} + \frac{y^2}{b^2} = 0$ | $\sin 0° = 0$, $\cos 0° = 1$

or, $\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$

or, $\frac{x}{a} - \frac{y}{b} = 0$

or, $\frac{x}{a} = \frac{y}{b}$

$\therefore \boxed{y = \frac{b}{a}x}$ ($= \tan\theta\, x$) $\Longrightarrow$ Eq^n. of a St. Line

So, the emergent ray will be plane-polarised.

$\boxed{\tan\theta = \frac{b}{a}} \rightarrow$ Slope of the St. Line.

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Double Refraction (2)

Erasmus Bartholinus discovered in 1669 that when an ordinary light ray of unpolarised wave is made to incident on crystals like calcite, Quartz etc., It is refracted into two rays. One of these rays obeys the laws of refraction, called ordinary ray (O-ray). While other does not obey. This ray is called extraordinary ray (E-Ray). Both the O-Ray and E-ray are plane polarised.

This phenomenon, in which a single incident ray is refracted into two rays is called Double refraction or Birefringence.

This Phenomenon is exhibited by other crystals also. But It is to be noted that, the crystals, which have cubic arrangement of atoms can not show double refraction.

✓ Optic Axis

Optic axis of a crystal is defined as the direction through it, along which if a ray travels then there will be no double refraction of the ray. Both the O-ray & E-ray move with equal velocity.

If the ray is allowed to pass through any other direction, except optic axis, the ray will be divided into O-Ray and E-ray and vel. of the two rays will be different.