25.

If N is the number of dipoles per unit volume then magnetisation, M, per unit vol^m will be

$$M = N\mu = M_s .$$

where M_s is the saturation value of magnetisation

Case II :- When $\mu H \ll K_B T$: This means α is small i.e. temperature T is high or H is small. We find that

$$|L(\alpha)|_{\alpha \to 0} = \left| \frac{1+e^{-2\alpha}}{1-e^{-2\alpha}} - \frac{1}{\alpha} \right|_{\alpha \to 0}$$

$$= \left| \frac{e^{\alpha}+e^{-\alpha}}{e^{\alpha}-e^{-\alpha}} - \frac{1}{\alpha} \right|_{\alpha \to 0}$$

$$= \left| \frac{2+\frac{2\alpha^2}{2!}}{2\alpha+\frac{2\alpha^3}{3!}} - \frac{1}{\alpha} \right|_{\alpha \to 0}$$

$$= \left| \frac{1}{\alpha}\left(1+\frac{\alpha^2}{2}\right)\left(1-\frac{\alpha^2}{6}\right) - \frac{1}{\alpha} \right|_{\alpha \to 0}$$

$$= \frac{\alpha}{3} = \frac{\mu H}{3 K_B T},$$

so that

$$M = \frac{N\mu^2 H}{3 K_B T} . \longrightarrow (1)$$

If n_0 is the total number of dipoles then the number of dipoles, n, inclined at an angle θ with the field direction is given by

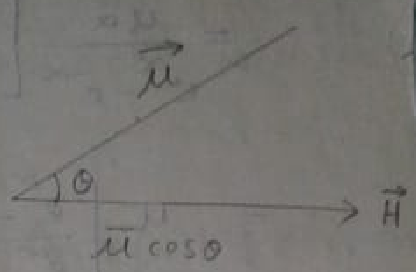

Fig. 1 Showing the resolved component of magnetic moment in the field direction

$$n_0 = n_0 \exp\left[\mu(H\cos\theta/K_BT)\right]$$

Therefore average magnetic moment $\bar{\mu}$ in the field direction will be obtained by dividing the sum of the resolved components of the magnetic moments of all the dipoles in the field direction by the total number of dipoles, i.e.,

$$\bar{\mu} = \frac{\int \mu\cos\theta\, dn}{\int dn}$$

$$= \frac{\int_0^\pi \mu\cos\theta \exp(\mu H\cos\theta/K_BT)\sin\theta\, d\theta}{\int \exp[(\mu H\cos\theta/K_BT)\sin\theta\, d\theta]}$$

Putting $\alpha = \dfrac{\mu H}{K_BT}$, we can write the above expression as

$$\bar{\mu} = \mu\frac{d}{d\alpha}\log_e\int_0^\pi \exp[(\alpha\cos\theta)\sin\theta\, d\theta]$$

$$= \mu\frac{d}{d\alpha}\log_e\frac{e^{\alpha} - e^{-\alpha}}{\alpha}$$

$$= \frac{\mu \alpha}{e^{\alpha} - e^{-\alpha}} \left[\frac{e^{\alpha} + e^{-\alpha}}{\alpha} - \frac{e^{\alpha} - e^{-\alpha}}{\alpha^2} \right]$$

$$= \mu \left[\frac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}} - \frac{1}{\alpha} \right]$$

$$= \mu \left[\coth \alpha - \frac{1}{\alpha} \right]$$

$$= \mu L(\alpha), \longrightarrow (1)$$

where $L(\alpha)$ is called Langevin function.

Case I:— When $\mu H \gg K_B T$: This means

$$\alpha = \frac{\mu H}{K_B T}$$

will be quite large. For such a case temperature T is small or H is large. We find that

$$|L(\alpha)|_{\alpha \to \infty} = \left| \coth \alpha - \frac{1}{\alpha} \right|_{\alpha \to \infty}$$

$$= \left[\frac{1 + e^{-2\alpha}}{1 - e^{-2\alpha}} - \frac{1}{\alpha} \right]_{\alpha \to \infty}$$

that is, Langevin function tends to unity. From eq^n (1), then

$$\bar{\mu} = \mu.$$

Classical Theories of Paramagnetism:- Imp

Langevin's Theory:-

Langevin considered a paramagnetic gas in which each atom or molecule possesses a permanent magnetic moment. The mutual magnetic interaction between the different gas particles is neglected. When gas is subjected to an external magnetic field, the state of magnetization will be determined by the two factors:-

(a) The applied magnetic field:- which tends to align the magnetic axes of molecules (that are considered as permanent magnetic dipoles) in its own direction.

(b) The thermal agitation: that works to disorganise the orderly state so produced.

At any given temp^r, a kind of statistical equilibrium is reached with majority of molecules having their axes parallel to the field direction and hence contribute to the intensity of magnetisation.

The probability that a dipole is inclined at an angle θ to the field direction in thermal equilibrium is proportional to

$$\exp.\left[\mu\left(H\cos\theta/K_B T\right)\right].$$

This gives the magnetic susceptibility,

$$\chi = \frac{M}{H} = \frac{N\mu^2}{3K_BT} \longrightarrow (2)$$

μ is of the order of one Bohr magneton ($\simeq 10^{-20}$ erg / gauss) so that for a field of 10^4 gausses, $\mu H \simeq 10^{-16}$ erg. At room temperature, $\frac{K_B T}{3} \cong 10^{-14}$ erg. Therefore condition $\mu H \ll K_B T$ is satisfied except for very low temperatures. Relation for susceptibility can also be written as

$$\chi = \frac{\text{Constant}}{T}, \longrightarrow (3)$$

and is called Curie law.

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