Maximisation of Tax Revenue in a Competitive market.

Let us consider demand and supply functions in a competitive market

$$Q_d = a - bP$$
$$Q_s = -c + dP$$
$$Q_d = Q_s.$$

When the Govt. imposes excise duty of 't' per unit of output, the supply function depend on pre-tax Price i.e, $(P-t)$. Thus

$$Q_s = -c + d(P-t).$$

We know in eqm $Q_d = Q_s$. Thus,

$$a - bP = -c + dP - dt.$$
$$-bP - dP = -c - a - dt$$
$$+(bP + dP) = +(c + a + dt)$$
$$P(b+d) = c + a + dt$$
$$\therefore P = \frac{c + a + dt}{b+d}.$$

Now substituting $P = \frac{c+a+dt}{b+d}$ in demand fun we get,

$$Q_d = a - b\left[\frac{c+a+dt}{b+d}\right].$$
$$= \frac{\cancel{ab} + ad - bc - \cancel{ab} - bdt}{b+d}$$
$$= \frac{ad - bc - bdt}{b+d}.$$

We know the tax revenue function is

$$T = tQ = t\left[\frac{ad - bc - bdt}{b+d}\right].$$

$$= \frac{adt - bct - bdt^2}{b+d}$$

Maximisation of tax revenue requires that

$$\frac{dT}{dt} = 0 \text{ and } \frac{d^2T}{dt^2} < 0.$$

$$\frac{dT}{dt} = \frac{ad - bc - 2bdt}{b+d} = 0.$$

$$\Rightarrow +2bdt = +(ad - bc)$$

$$\therefore t = \frac{ad - bc}{2bd}$$

The second order condition of T w.r.t. t

$$\frac{d^2T}{dt^2} = \frac{-2bd}{b+d} < 0.$$

Hence the tax revenue maximising rate of tax is $t = \dfrac{ad - bc}{2bd}$

Given the competitive market of the following form:

$Qd = 16 - 2P$
$Qs = -4 + 2P$

$Qd = Qs.$

If the govt. imposes a tax of t per unit of output, find the value of t for which the tax revenue will be maximum and also price per unit of output.

The supply function after imposition of tax will be,

$Qs = -4 + 2(P - t).$
$Qs = -4 + 2P - 2t.$

The equm condition is $Qd = Qs.$

$$16 - 2P = -4 + 2P - 2t.$$
$$-2P - 2P = -4 - 16 - 2t.$$
$$+(2P + 2P) = +(20 + 2t)$$
$$4P = 20 + 2t$$
$$\therefore P = \frac{20 + 2t}{4}.$$

Now substituting $P = \frac{20+2t}{4}$ in demand function the equm quantity can be obtained

$$Q = 16 - 2\left(\frac{20+2t}{4}\right)$$
$$= 16 - \frac{40 - 4t}{4}$$
$$= \frac{64 - 40 - 4t}{4}$$
$$= 16 - 10 - t = 6 - t.$$

Total tax revenue is given by

$$T = tQ = t(6-t)$$
$$= 6t - t^2.$$

The maximisation of tax revenue requires that $\frac{dT}{dt} = 0$ and $\frac{d^2T}{dt^2} < 0$.

$$\frac{dT}{dt} = 6 - 2t = 0.$$
$$\Rightarrow \quad t = 3.$$
$$\frac{d^2T}{dt^2} = -2 < 0.$$

Therefore, the tax revenue will be maximum when the rate of tax is 3 per unit of output.

we have $P = \frac{20+2t}{4}$

$$= \frac{20+2(3)}{4}$$
$$= \frac{26}{4} = 6.5$$

$Q_d = 20 - 3P$
$Q_s = -10 + 2P$
$Q_d = Q_s$, tax 't' per unit of output.

$\bar{t} = 12$, $\bar{Q} = 1.5$, $\bar{P} = 17$ and $\bar{\pi} = 15$

$Q_d = 18 - 2P$
$Q_s = -7 + 3P$
$Q_d = Q_s$.
tax 't' per unit of output.

$$\bar{t} = \frac{10}{12}$$