Sagitta Theorem

Consider a chord DEF of an arc DAF of radius R (= AC) where E is the middle point of DF as shown in the fig. AE is called sagitta of arc DAF.

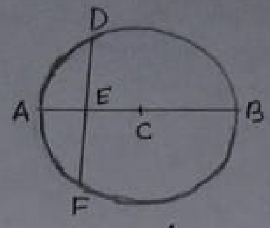

Fig.

By geometry of the circle,

$$DE^2 = AE \times EB = AE(AB - AE)$$
$$= AE \times AB - AE^2$$

$$\therefore \quad DE^2 = AE \times AB - AE^2$$

Neglecting AE^2 being very small,

$$\therefore \quad DE^2 = AE \times AB$$

$$\text{or,} \quad AE = \frac{DE^2}{AB} = \frac{DE^2}{2R}$$

Thus, sagitta of an arc varies inversely as its radius of curvature. This statement is termed as sagitta theorem.

respectively (RR_1 or $R'R_1' < AP$) such that

$$AP = V_1 t \quad \text{or,} \quad \frac{AP}{V_1} = t$$

and $RR_1 = R'R_1' = V_2 t$

$$\text{or,} \quad \frac{RR_1}{V_2} = t$$

$$\therefore \quad \frac{AP}{V_1} = \frac{RR_1}{V_2}$$

$$\text{or,} \quad RR_1 = \frac{V_2}{V_1} AP$$

$$= \frac{\mu_1}{\mu_2} AP \longrightarrow (1)$$

Where V_1 and V_2 be the velocities in the 1st medium of R.I. μ_1 and 2nd medium of R.I. μ_2. respectively.

Draw RL and R_1M perpendiculars from R and R_1 ~~upon~~ upon OP.

AL, PM and PL respectively are the sagittas of curves RAR', R_1PR_1' and RPR'. Using ~~sat~~ sagitta's Theorem, we have,

$$AL = \frac{y^2}{2u}$$

$$PM = \frac{y^2}{2v}$$

$$PL = \frac{y^2}{2R}$$

From the fig.

$RL = R_1M = y$

$OP = u$

$IP = v$

From the fig. 1. it is clear that

$$PM = PL - LM$$

$$= PL - RR_1$$

Substituting for RR_1 from eqn. (1) $\Rightarrow$

Formula for Thin double concave Lens:

Let O be the point object situated on the principal axis of a double concave lens as shown in the fig. 3. XAY is the position of incident wave-front. During the time of secondary disturbance travels from X to X′ (through the material of the lens) the disturbance from A reaches B.

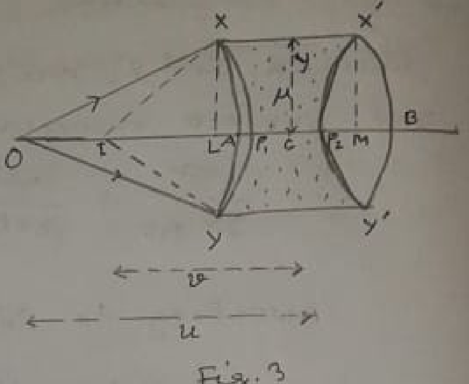

Fig. 3

$$\therefore \frac{XX'}{V_2} = \frac{AP_1}{V_1} + \frac{P_1P_2}{V_2} + \frac{P_2B}{V_1}$$

[where V_1 be the velocity through air medium and V_2 be the velocity through the concave lens.

Draw XL and X′M ⊥rs to the principal axis.

But $XX' = LM = LP_1 + P_1P_2 + P_2M$ [from fig. 3]

$$\therefore \frac{LP_1 + P_1P_2 + P_2M}{V_2} = \frac{AP_1}{V_1} + \frac{P_1P_2}{V_2} + \frac{P_2B}{V_1}$$

or, $$\frac{LP_1}{V_2} + \frac{P_1P_2}{V_2} + \frac{P_2M}{V_2} = \frac{AP_1}{V_1} + \frac{P_1P_2}{V_2} + \frac{P_2B}{V_1}$$

or, $$\frac{1}{V_1}(AP_1 + P_2B) = \frac{1}{V_2}(LP_1 + P_2M)$$

$$\therefore AP_1 + P_2B = \frac{V_1}{V_2}(LP_1 + P_2M) \longrightarrow (1)$$

Since $AP_1 = P_1L - AL$ and $P_2B = P_2M + MB$

∴ From eqn. (1) ⇒

$$P_1L - AL + P_2M + MB = \mu(LP_1 + P_2M) \longrightarrow (2)$$

[Since $\frac{\mu_2}{\mu_1} = \frac{V_1}{V_2}$, for air medium, $\mu_1 = 1$

μ is the refractive index of material of concave lens. i.e., $\mu_2 = \mu$

$$\therefore \frac{V_1}{V_2} = \mu \;]$$

Huygen's Wave Theory:

According to Huygen's Theory, "Light is sort of disturbance. The particle of the medium vibrate in a direction, at right angles to the direction of propagation of disturbance. The process is called wave-motion."

Considering light to be a wave-motion, the following properties of light could be explained:

(1) Rectilinear propagation of light. (2) Reflection (3) Refraction (4) Interference (5) Diffraction (6) Polarization.

But photo-electric effect could not be explained on the basis of this Theory.

Formula for Refraction at a Spherical Surface:

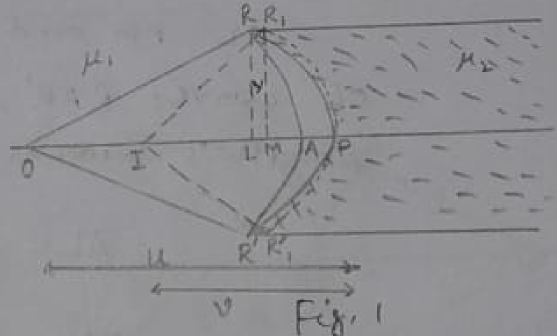

Fig. 1

Let RR' be the section of single refracting surface ⊥r to the plane of the paper as shown in the fig. 1. An incident wave starting from a point object O approaches the interface separating the two media of R. I. μ_1 and μ_2 ($\mu_2 > \mu_1$). Let RAR' be the position of incident wave front at any instant of time. Applying Huygen's principle to the wave front at this instant. During this time, secondary disturbance from A to P, that from R to R_1 and R' to R_1'

From eqⁿ ② ⇒

$$-AL + MB = \mu(LP_1 + P_2M) - (P_1L + P_2M)$$

$$= (\mu - 1)(LP_1 + P_2M) \rightarrow ③$$

According to Sagitta Theorem,

$$AL = \frac{y^2}{2 \times OA}, \quad MB = \frac{y^2}{2 \times BI}, \quad P_1L = \frac{y^2}{2 \times R_1}, \quad P_2M = \frac{y^2}{2 \times R_2}$$

Substituting the values in eqⁿ ③, we get

$$-\frac{y^2}{2 \times OA} + \frac{y^2}{2 \times BI} = (\mu - 1)\left[\frac{y^2}{2 \times R_1} + \frac{y^2}{2 \times R_2}\right]$$

or, $$-\frac{1}{OA} + \frac{1}{BI} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

According to sign conventions,

$$OA = -u, \quad BI = -v$$

$$R_1 = -ve, \quad R_2 = +ve$$

$$\therefore -\frac{1}{-u} + \frac{1}{-v} = (\mu - 1)\left[\frac{1}{-R_1} + \frac{1}{R_2}\right]$$

or, $$-\left(\frac{1}{v} - \frac{1}{u}\right) = -(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or, $$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

This is the Lens formula for double concave lens.

For rays coming from infinity

i.e. $u = \infty, \quad v = f$

$$\therefore \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$PM = PL - \frac{\mu_1}{\mu_2} \times AP = PL - \frac{\mu_1}{\mu_2}(PL - AL)$$

or, $\mu_2 PM = \mu_2 PL - \mu_1 (PL - AL)$

or, $\mu_2 PM = \mu_2 PL - \mu_1 PL + \mu_1 AL$

or, $\mu_2 PM - \mu_1 AL = (\mu_2 - \mu_1) PL$

Substituting for PM, AL and PL, we get,

$$\mu_2 \times \frac{y^2}{2v} - \mu_1 \times \frac{y^2}{2u} = (\mu_2 - \mu_1) \times \frac{y^2}{2R}$$

or, $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

This is the condition for refraction at a single surface as obtained in geometrical optics.

Formula for Thin double Convex Lens:

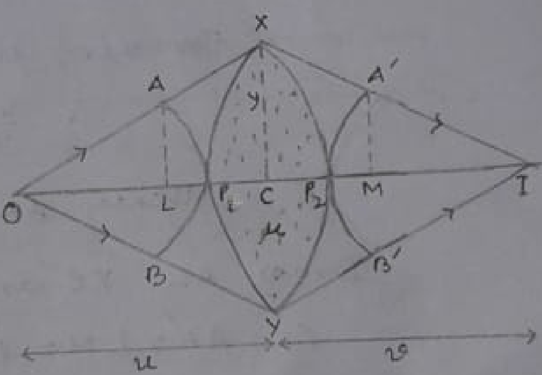

Fig. 2

Consider a double convex lens of R.I. μ and radii of curvature of the two surface as R_1 and R_2 respectively (Fig. 2). O be the point object situated on the principal axis of the Lens. AB is the incident wavefront which touches the Lens at its pole P_1. Secondary wavelets start from all the points of wave front AP_1B. As the light goes from P_1 to P_2, through the material of medium, disturbance from A and B reaches A' and B' respectively. So, $A'P_2B'$ gives the refracted wave-front, thus producing an image at I.

As the lens is thin, AXA' and BYB' can be assumed to be straight ~~ten~~ lines parallel to the principal axis.

Since the time for light in going from P_1 to P_2 is same as that of going from A to A' (or from B to B')

$$\therefore \quad \frac{P_1P_2}{V_2} = \frac{AA'}{V_1} = \frac{BB'}{V_1}$$

Where V_2 be the velocity of light passing through the Lens (R.I. μ) and v_1 be the velocity of light through air medium.

$$\text{or,} \quad AA' = \frac{V_1}{V_2} P_1P_2$$

$$\text{or,} \quad AA' = \frac{\mu_2}{\mu_1} P_1P_2$$

For air, $\mu_1 = 1$

$$\therefore \quad \frac{V_1}{V_2} = \mu$$

$$\frac{V_1}{V_2} = \frac{\mu_2}{\mu_1}$$

For air, $\mu_1 = 1$

$$\therefore \frac{V_1}{V_2} = \mu$$

$$\text{Hence} \quad AA' = \mu P_1P_2 \longrightarrow (1)$$

Draw AL, XC and $A'M$ $\perp$rs to the principal axis.

$$\therefore \quad AA' = LM = LP_1 + P_1P_2 + P_2M$$

Substituting in eq^n. (1) $\Rightarrow$

$$LP_1 + P_1P_2 + P_2M = \mu P_1P_2$$

$$\text{or,} \quad LP_1 + P_2M = (\mu - 1) P_1P_2$$

$$= (\mu-1)(P_1C + CP_2) \longrightarrow (2)$$

According to Sagitta's Theorem

$LP_1 = \frac{y^2}{2 \times P_1O}$, $P_2M = \frac{y^2}{2 \times P_2I}$, $P_1C = \frac{y^2}{2R_1}$, $CP_2 = \frac{y^2}{2R_2}$

where R_1 and R_2 are radii of curvature of 1st and 2nd surface respectively. From eqn ② ⟹

$$\therefore \frac{y^2}{2 \times P_1O} + \frac{y^2}{2 \times P_2I} = (\mu - 1)\left[\frac{y^2}{2R_1} + \frac{y^2}{2R_2}\right]$$

According to sign conventions,

$P_1O = -u$, $P_2I = +v$

$R_1 = +ve$, $R_2 = -ve$

$$\therefore \frac{y^2}{2(-u)} + \frac{y^2}{2(v)} = (\mu-1)\left[\frac{y^2}{2R_1} - \frac{y^2}{2R_2}\right]$$

$$\text{or, } -\frac{1}{u} + \frac{1}{v} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\text{or, } \frac{1}{v} - \frac{1}{u} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \longrightarrow ③$$

This is the lens formula for double convex lens.

If the incident beam comes from infinity,

i.e, $u = \infty$, $v = f$

$$\therefore \frac{1}{f} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$