## Chebyshev's Inequality :-

Statement :- If $x$ is a random variable with mean $\mu$ and variance $\sigma^2$ Then

$$Pr\{|x-\mu| \le \sigma t\} > 1 - 1/t^2 \longrightarrow (1)$$

Let $u = (x-\mu)^2$

$\Rightarrow E(U) = E(x-\mu)^2$ ; $a = \sigma^2$

$\because E(U) = a$ and $E\{(x-\mu)\}^2 = E\{x - E(x)\}^2 = \sigma^2$

Now from eq$^n$ (1) we get

$$P\{(x-\mu)^2 \le \sigma^2 t^2\} > 1 - 1/t^2$$

$$\Rightarrow P\{|x-\mu| \le \sigma t\} > 1 - 1/t^2 \qquad \because \sigma \text{ and } t \text{ are} > 0.$$

## Chebyshev's Lemma :-

if $U$ is a random variable which takes the Positive values only with $E(x) = a$, Then

$$P(U > at^2) < 1/t^2 \; ; \; t > 0.$$

Proof :- Let $u$ be a random variable which takes the Positive values $u_1, u_2, \ldots u_\alpha, u_{\alpha+1} \ldots u_n$ with respective Probabilities $p_1, p_2, \ldots p_\alpha, p_{\alpha+1}, \ldots p_n$. Let the values of $U$ be arrearged in descending order of magnitude ie $u_1 > u_2 > \ldots u_\alpha > u_{\alpha+1} > \ldots > u_n$. [Let $u_\alpha$ be the least value which is greater than $at^2$

Now by defination

$$E(u) = \sum_{i=1}^{n} u_i p_i = u_1 p_1 + u_2 p_2 + \ldots + u_n p_n$$

$$\Rightarrow a \ge u_1 p_1 + u_2 p_2 + \ldots + u_n p_n$$

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$\because E(U) = a$ and $u_\alpha p_1 + u_{\alpha+1} p_{\alpha+1} + \cdots + u_n p_n$

$\Rightarrow a \geq at^2 p_1 + at^2 p_2 + \cdots + at^2 p_\alpha$

[ $\because u_\alpha > at^2$ $\therefore u_1 > at^2, u_2 > at^2$

$\cdots$ ~~$u_{\alpha+1}$ at$^2$~~ $\cdots u_{\alpha+1} > at^2$ as

$u_1 > u_2 > \cdots > u_\alpha$. Therefore equality

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$\Rightarrow \frac{1}{t^2} > p_1 + p_2 + \cdots + p_\alpha$

$\Rightarrow \frac{1}{t^2} > P(U > at^2)$

$\Rightarrow P(U > at^2) < 1/t^2$.

Complementary Probability :-

Since $P(U > at^2) + P(U \leq at^2) = 1$.

$\Rightarrow P(U \leq at^2) = 1 - P(U > at^2) = 1 - 1/t^2$

$= 1 - 1/t^2 \mid \because P(U > at^2) < 1/t^2$

$\Rightarrow P(U \leq at^2) = 1 - 1/t^2$.

## Bienayme-chebyshev's Inequality:-

Let $g(x)$ be a ~~non neg~~ non-negative function of a random variable $x$.

Then for every $K > 0$, we get

$$P\{g(x) > K\} \leq \frac{E[g(x)]}{K} \longrightarrow (1)$$

Proof:-

We know that if $U$ is a random variable which takes positive values only with $E(U) = a$ then the chebyshev's lemma is given by

$$P\{(U > at^{r})\} \leq 1/t^{r}$$

Now let $U = g(x)$

$$\therefore E(U) = E\{g(x)\} = a.$$

Let $K = at^{2} \Rightarrow \frac{1}{t} = \frac{a}{K} = \frac{E\{g(x)\}}{K}$

From (1) we get

$$P\{g(x) > K\} \leq \frac{E\{g(x)\}}{K}.$$

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## Weak Law of Large Numbers:-

Statement:- Let $x_1, x_2, \ldots x_n$ be $n$ random variables with respected Expections, $a_1, a_2, \ldots$ respectively.

if (i) $E(x_1+x_2+\cdots+x_n)$ is finite

(ii) $V(x_1+x_2+\cdots+x_n) = D_n$ (say) exists

and (iii) $\lim_{n\to\infty} \frac{D_n}{n} \to 0$ Then

$$P\left\{\left|\frac{x_1+x_2+\cdots+x_n}{n} - \frac{a_1+a_2+\cdots+a_n}{n}\right| < \epsilon\right\}$$

as $n\to\infty$, $\epsilon$ is a very small positive number.

Proof:- From chebyshev's Lemma we have, $U$ is a random variable which takes only Positive values only, $t>0$ and $E(U)=a$ Then

$$P\{U \le at^2\} > 1 - 1/t^2 \longrightarrow (1)$$

Let $U = \{(x_1+x_2+\cdots+x_n) - (a_1+a_2+\cdots+a_n)\}^2$

$$\Rightarrow E(U) = E\{(x_1+x_2+\cdots+x_n) - (a_1+a_2+\cdots+a_n)\}^2$$

$$\Rightarrow a = V(x_1+x_2+\cdots+x_n)$$

$$\Rightarrow a = D_n$$

Now from eqn no (1) we get

$$P\left[\{(x_1+x_2+\cdots+x_n) - (a_1+a_2+\cdots+a_n)\}^2 \le D_n t^2\right] > 1-$$