$1+x^{-}$

2.5 How to find the Principal Value of a given inverse trigonometric function ?

(1) To find the principal value of $\sin^{-1} x$

Method : (*i*) Let $y = \sin^{-1} x$ so that $\sin y = x$ where $x \in \mathbf{R}$.

(*ii*) (*a*) Then $\sin y = x = \sin \theta$

(say; θ is known)

$$\text{Now } \sin^{-1} : [-1, 1] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

Therefore, the principal value branch of $\sin^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \sin y = \sin \theta \Rightarrow y = \theta$; where θ lies in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

(*b*) If $\sin y = -\sin \theta \Rightarrow \sin y = \sin(\pi - \theta)$

so that the required principal value is given by

$$y = \pi - \theta, \text{ where } \frac{\pi}{2} < \theta < \frac{3\pi}{2}$$

(2) Proceeding as above, we can find the principal values of other inverse trigonometric functions, keeping in mind the principal value branch of the given function.

(3) We can also find the principal values of $\text{cosec}^{-1} x$, $\sec^{-1} x$ and $\cot^{-1} x$ by expressing cosec y, sec y and cot y as $\frac{1}{\sin y}$, $\frac{1}{\cos y}$ and $\frac{1}{\tan y}$ respectively, and then solving for $\sin y = \sin \theta$ *etc.*