

3rd Semester  
Home assignment  
Paper 3026  
Marks - 50

Answer any five

(1) (a) Write out a complete Cayley table for  $D_3$ . Is  $D_3$  Abelian?

(b) In  $D_4$  find all elements  $x$  such that

(i)  $x^3 = V$

(ii)  $x^3 = R_{90}$

(iii)  $x^3 = R_0$

(iv)  $x^2 = H$ .

(2) For all integers  $n \geq 1$ , the set of complex  $n$ th roots of unity

$$\left\{ \cos \frac{k \cdot 360^\circ}{n} + i \sin \frac{k \cdot 360^\circ}{n} \mid k = 0, 1, \dots, n-1 \right\}$$

is a group under multiplication.

(3) (a) For group elements  $a$  and  $b$ , prove that  $(ab)^{-1} = b^{-1}a^{-1}$ .

(b) Let  $G$  be a group and let  $H = \{a^{-1} \mid a \in G\}$ . Show that  $G = H$ .

(c) Prove that in a group  $(a^{-1})^{-1} = a$  for all  $a$ .

(4) (a) Let  $G$  be a group and let  $a \in G$ .

Prove that  $\langle a \rangle = \langle a^{-1} \rangle$

(b) Show that  $\langle (14) \rangle = \langle (3) \rangle = \langle (5) \rangle$ .

or  $\langle (14) \rangle = \langle (1) \rangle$ ?

(5) (a) Find an generator of

$Z_6, Z_8$  and  $Z_{20}$

(b) Every subgroup of a cyclic group is cyclic.

(c) Find all subgroups of  $Z_{30}$ .

6. (a) The set of even permutations

in  $S_n$  form a subgroup of  $S_n$ .

(b) Determine whether the following

Permutation are even or odd

(i)  $(135)$  (ii)  $(1356)$  (iii)  $(13567)$

(iv)  $(12)$  (v)  $(134)$  (vi)  $(152)$  (vii)  $(1243)$  (viii)  $(3521)$

7 (a) State and Prove the First Isomorphism Theorem

(b) Let  $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \mid a, b, d \in \mathbb{R}, ad \neq 0 \right\}$  is

A normal subgroup of  $GL(2, \mathbb{R})$

(c) State and Prove the Lagrange's Theorem.