3rd Semester

Home assignment

Paper 3016

Marks – 50

Answer any five

(1) (a) Let I be an interval in $\mathbb{R}$, let $f : I \to \mathbb{R}$ and $c \in I$. Suppose there exists constant K and L such that $|f(x) - L| \leq K|x - c|$ for $x \in I$. Show that $\lim_{x \to c} f(x) = L$

(b) Determine a condition on $|x-4|$ that will assure that $|\sqrt{x} - 2| < \frac{1}{2}$

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(x+y) = f(x) + f(y)$ for all x, y in $\mathbb{R}$. Assume that $\lim_{x \to 0} f = L$ exists. Prove that $L = 0$ and then prove that f has a limit at every point $c \in \mathbb{R}$.

3) (a) Let f be defined for all $x \in \mathbb{R}$, $x \neq 2$ by $f(x) = \dfrac{x^2 + x - 6}{x - 2}$. Can f be defined at $x = 2$ in such a way that f is continuous at this point?

(b) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and that $f(r) = 0$ for every rational number r. Prove that $f(x) = 0$ for all $x \in \mathbb{R}$

(4) State and Prove the Location of Root theorem

(5) (a) Show that the function $f(x) = \frac{1}{x^2}$ is uniformly continuous on $A = [1, \infty)$, but that it is not uniformly continuous on $B = (0, \infty)$

(b) Show that if f and g are uniformly continuous on a subset A of $\mathbb{R}$, then $f+g$ is uniformly continuous on A.

(6) (a) State and Prove the Rolle's Theorem.

(b) Determine where ~~each~~ of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is differentiable and find the derivative

$$f(x) = |x| + |x+1|$$

(7) Does there exist a function which is continuous at every point but whose derivative does not exist anywhere. Justify