The slope at point B,

$$\tan(\theta - \delta\theta) \simeq \theta - \delta\theta$$

$$= \frac{\partial y}{\delta x} - \frac{\delta^2 y}{\delta x^2}\delta x$$

The upward component of the tension at B $= T\tan(\theta - \delta\theta)$

$$= T(\theta - \delta\theta)$$

$$= T\left[\frac{\partial y}{\delta x} - \frac{\delta^2 y}{\delta x^2}\delta x\right] \longrightarrow (2)$$

The resultant downward tension [from eq^n (1) and (2)]

$$F = T\frac{\partial y}{\delta x} - T\left[\frac{\partial y}{\delta x} - \frac{\delta^2 y}{\delta x^2}\delta x\right]$$

$$= T\frac{\delta^2 y}{\delta x^2}\delta x \longrightarrow (3)$$

Let m be the mass of the string per unit length.

$\therefore$ Mass of the element AB $= m\,\delta x$

Acceleration of the element along y axis

$$= \frac{\delta^2 y}{\delta t^2}$$

$\therefore$ Force acting on the element, $F = m\,\delta x\,\frac{\delta^2 x}{\delta t^2} \longrightarrow (4)$

[F = ma]

From eq^n (3) and (4),

$$m\cancel{\delta x}\,\frac{\delta^2 x}{\delta t^2} = T\frac{\delta^2 y}{\delta x^2}\,\cancel{\delta x}$$

So, the volume enclosed with in A_1 to B_1

$$= (dx + d\xi) . 1$$

$$= dx + d\xi$$

Change in volume undergone by the original volume →

$$= (dx + d\xi) - dx$$

$$= d\xi$$

∴ Change in volume per unit volume,

$$= \frac{d\xi}{dx} = \text{volume strain}.$$

Now, bulk modulus, ~~stress~~

$$E = \frac{\text{Stress}}{\text{Volume strain}}$$

$$\Rightarrow \text{stress} = E \times \text{volume strain}$$

$$= E \times \frac{d\xi}{dx}$$

$$\Rightarrow \text{Pressure} = E \times \frac{d\xi}{dx}$$

If P be the pressure on A and $P + \frac{dp}{dx} \times dx$ be the pressure on B then the change in pressure =

$$P + \frac{dp}{dx} dx - P = \frac{dp}{dx} dx$$

$$\Rightarrow \frac{\delta^2 y}{\delta t^2} = \frac{T}{m}\frac{\delta^2 y}{\delta x^2} \longrightarrow (5)$$

This is similer to the differential equation of wave motion,

$$\frac{\delta^2 y}{\delta t^2} = v^2 \frac{\delta^2 y}{\delta x^2} \longrightarrow (6)$$

Compearing eq^n (5) and (6),

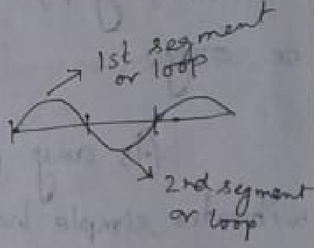
$$v^2 = \frac{T}{m}$$

$$\therefore v = \sqrt{\frac{T}{m}} \longrightarrow (7)$$

We know,

$v = \nu\lambda$, where, ν = frequency

If l be the length of the string for p segment then the length of each segment $= \frac{l}{p}$



$$\therefore \frac{l}{p} = \frac{\lambda}{2} \text{ (for one segment)}$$

$$\therefore \lambda = \frac{2l}{p}$$

$$\therefore v = \nu\left(\frac{2l}{p}\right) \longrightarrow (8)$$

From (7) and (8),

$$\sqrt{\frac{T}{m}} = \nu\left(\frac{2l}{p}\right)$$

$$\therefore \nu = \frac{p}{2l}\sqrt{T/m}$$

$$= \frac{d}{dx}\left(E\frac{d\xi}{dx}\right)dx$$

$$= E\frac{d^2\xi}{dx^2}dx$$

= Force per unit area.

$$= \rho dx \frac{d^2\xi}{dt^2}$$

$$P = \frac{F}{A}$$

$$= \frac{ma}{A}$$

$$= \frac{V\rho a}{A}$$

$$= dx\rho a$$

$$= \rho dx \frac{d^2\xi}{dt^2}$$

Hence, $E\frac{d^2\xi}{dx^2}dx = \rho dx \frac{d^2\xi}{dt^2}$

$$\therefore \frac{d^2\xi}{dt^2} = \frac{E}{\rho}\frac{d^2\xi}{dx^2} \longrightarrow (1)$$

We know,

the expression of differential equation of wave motion, $\frac{d^2\xi}{dt^2} = v^2\frac{d^2\xi}{dx^2} \longrightarrow (2)$

Comparing equation (1) and (2),

$$v^2 = \frac{E}{\rho}$$

$$\Rightarrow v = \sqrt{\frac{E}{\rho}}$$

Thus, the velocity of the sound in gaseous medium

$$v = \sqrt{\frac{E}{\rho}}$$

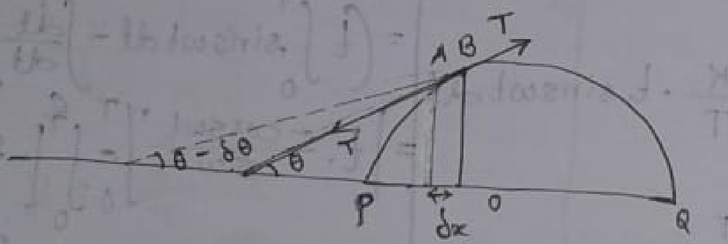
where, E = elasticity of the medium.

ρ = density of the medium.

Equation of transverse vibration of a stretched string.

or

Velocity of transverse vibration of a stretched string.



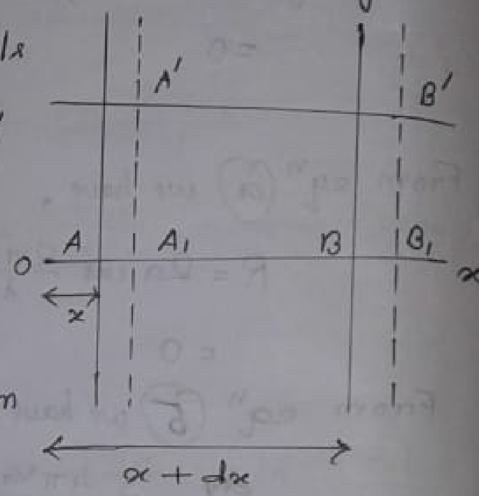
Consider a string PQ stretched by a tension T along x direction. Let the string is plucked at the centre O and let it free. So the string vibrates along y direction.

Consider a small element AB of length δx. The tangents drawn at A and B makes angles θ and $\theta-\delta\theta$

Sound waves

Velocity of Longitudinal waves in Gases (Fluid)

Imagine a longitudinal plane sound wave consisting condensation and rarefaction travells through a gas medium of density ρ to the right with velocity c.



Let Ox be the direction of advance of the wave and let AB be a very thin layer of the medium of unit cross section.

Let x be the distance of A from the origine at any instant and $x+dx$ be the distance of B from the origine. fig.

Hence the thickness of the slab $AB = dx$

The volume ~~enclo~~ enclosed with the slab $AB = dx.1$

Let ξ be the displacement of A to A_1 after time dt and $\xi + d\xi$ be the displacement of B to B_1.

Now, the distance between the phase,

$$A_1 \text{ to } B_1 = (x + dx + \xi + d\xi) - (x + \xi) = dx + d\xi$$

with the x axis.

The tension at A is resolve into two rectangular components. The downward component of tension at $A = T\sin\theta$.

If θ is small,

$$\therefore \sin\theta \simeq \theta$$

Again, $\theta \simeq \tan\theta$

Now, $\tan\theta = \frac{\delta y}{\delta x}$ at A (i.e slope at A)

$$\therefore \theta = \frac{\delta y}{\delta x}$$

$\therefore$ The downward component of tension at $A = T\tan\theta$

$$= T\frac{\delta y}{\delta x} \longrightarrow (1)$$

The rate of change of slope w.r.t the length of the element $= \frac{\delta}{\delta x}\left(\frac{\delta y}{\delta x}\right)$

$$= \frac{\delta^2 y}{\delta x^2}$$

Thus, the change in slope for a distance, δx

$$\delta\theta = \frac{\delta^2 y}{\delta x^2}\,\delta x$$