Equation of a plane progressive Harmonic Wave

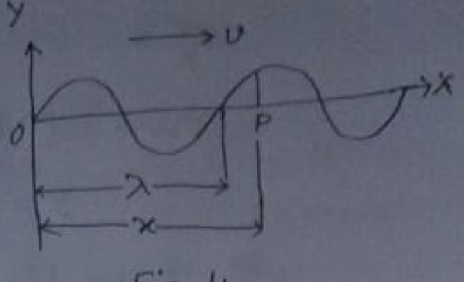

Fig. 4

If during ~~progr~~ propagation of wave in a medium, particles of medium executes SHM, Then the wave is said to be a Simple Harmonic progressive wave and if the amplitude of such wave remains unchanged, it is said to be simple Harmonic plane progressive wave.

A wave which travels in a medium continuously in the same direction without any change in its amplitude is called plane progressive wave.

A plane progressive wave may be transverse or Longitudinal in nature.

Let us consider a plane progressive wave travels along x-axis from the origin O as shown in the Fig. 4. The displacement (y) of a particle at any instant t is given by

$$y = A\sin\omega t \longrightarrow (1)$$

where A = Amplitude of the wave motion.

$\omega = \frac{2\pi}{T}$ is the angular velocity.

Let us find the displacement of ~~the~~ another particle P at a distance x from origin O at any time t. A particle on +ve x-axis recieves a disturbance, at definite time later than that proceeding it. ∴ the phase lag (phase difference) of particles w.r.t. at O goes on increasing as we move more and more ~~...~~ away from point O. If ~~φ~~ ϕ is the phase diff. of particle P w.r.t. particle at O, then the displacement of particle at P at any instant t is given by

$$y = A\sin(\omega t - \phi) \longrightarrow (2)$$

where, ω = angular velocity

K = the wave no. (propagation constant)

Since $\omega = 2\pi n$

and $K = \frac{2\pi}{\lambda}$, where λ = wavelength.

$\therefore$ wave velocity

$$v = \frac{\omega}{K}$$

$$= 2\pi n \times \frac{\lambda}{2\pi}$$

$$= n\lambda$$

$\therefore$ wave velocity = frequency × wavelength.

Thus, the wave velocity is the velocity at which a plane wave advances with constant phase $(\omega t - Kx)$.

If phase is constant, then

$$\frac{d}{dt}(\text{phase}) = 0$$

$$\text{or, } \frac{d}{dt}(\omega t - Kx) = 0$$

$$\text{or, } \omega - K\frac{dx}{dt} = 0$$

$$\text{or, } K\frac{dx}{dt} = \omega$$

$$\therefore \frac{dx}{dt} = \frac{\omega}{K}$$

The term $\frac{dx}{dt}$ is wave velocity (v)

$$\therefore v = \frac{\omega}{K} = \frac{dx}{dt}.$$

Since $K = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

$$\therefore \frac{\omega}{K} = \frac{2\pi}{T} \times \frac{\lambda}{2\pi} = \frac{\lambda}{T} = n\lambda = v$$

From eqn. 7

$$y = A\sin(\omega t - Kx) \longrightarrow (8) \quad \left| \begin{array}{l} \frac{\omega}{K} = v \\ \therefore Kv = \omega \end{array} \right.$$

The exponential form of eqn. 8 is given by

$$y = Ae^{i(\omega t - Kx)} \longrightarrow (9)$$

where $i^2 = -1$

Equations 8 and 9 also represent the eqn. of plane progressive wave ~~progd~~ propagating along +ve x-axis. If the wave propagates along -ve x-axis, then only put $-x$ ~~in~~ for x.

If ϕ_0 is the initial phase, then the equation of plane progressive wave along +ve x-axis

$$y = A\sin[(\omega t - Kx) + \phi_0] \longrightarrow (10).$$

Wave Velocity:

The velocity with which the wave propagates through the medium is called wave velocity. As the wave propagates the crest and trough (Compressions and rarefactions) travel with wave velocity. Since, all the particles at the crest or at the trough are in same phase, we can say that the phase travels with the wave ~~velocity~~. velocity. Hence, wave velocity is also called phase velocity.

The wave velocity (phase velocity) through a given medium is constant. It depends on the nature of the ~~medium~~ medium alone. The wave velocity is independent of time and also independent of source producing the waves.

Mathematically the wave velocity is given by

$$v = \frac{\omega}{K}$$

Since for a distance λ, phase changes by 2π, the change in phase for distance x, ~~will be~~ will be $\frac{2\pi}{\lambda}x$. i.e., $\phi = \frac{2\pi}{\lambda}x$

From eq^n (2), we get

$$y = A \sin\left(\omega t - \frac{2\pi}{\lambda}x\right) \longrightarrow (3)$$

If T be the time period of vibration, then

$$\omega = \frac{2\pi}{T}$$

$$(3) \Longrightarrow \quad y = A \sin\left(\frac{2\pi}{T}.t - \frac{2\pi}{\lambda}x\right)$$

$$= A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \longrightarrow (4)$$

We know, $v = n\lambda$ (n = frequency)

$$= \frac{\lambda}{T}$$

$$\text{or, } \frac{1}{T} = \frac{v}{\lambda}$$

From eq^n. (4) $\Rightarrow$

$$y = A \sin 2\pi\left(\frac{v}{\lambda}t - \frac{x}{\lambda}\right)$$

$$= A \sin \frac{2\pi}{\lambda}(vt - x) \longrightarrow (5)$$

Equation (4) and (5) are called plane progressive wave equations propagative along +ve x-axis. For -ve x-axis

$$y = A \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \text{———} (6a)$$

~~and y = A Sin 2π(t/T +~~

$$\text{and } y = A \sin \frac{2\pi}{\lambda}(vt + x) \longrightarrow (6b)$$

The quantity, $\frac{2\pi}{\lambda} = K$ called propagation constant.

Eq^n. (5) becomes

$$y = A \sin K(vt - x)$$

$$= A \sin(Kvt - Kx) \longrightarrow (7)$$

Particle Velocity and Acceleration

The individual particles which makes up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as particle velocity.

The eq^n. of a plane progressive wave is

$$y = A \sin(\omega t - Kx) \longrightarrow (1)$$

The velocity of any particle in its vibratory motion is obtained by differentiating this eq^n. w.r.t. time t keeping x constant.

$$\therefore \frac{dy}{dt} = u$$

$$\text{or,} \quad u = \frac{d}{dt}\left[A \sin(\omega t - Kx)\right]$$

$$= A \cos(\omega t - Kx) \cdot \omega$$

$$= A\omega \cos(\omega t - Kx) \longrightarrow (2)$$

The maximum value of $\cos(\omega t - Kx) = 1$

Hence maximum particle velocity

$$u_{max} = A\omega \underbrace{\cancel{\cos(\omega t - Kx)}}_{\cancel{1}} \cdot 1$$

$$= A\omega \longrightarrow (3)$$

The acceleration of any particle at any time in its vibratory motion is given by

$$a = \frac{du}{dt} = \frac{d}{dt}\left[A\omega \cos(\omega t - Kx)\right]$$

$$= -A\omega \sin(\omega t - Kx) \cdot \omega$$

$$= -A\omega^2 \sin(\omega t - Kx)$$

$$= \cancel{A\omega^2 y} - \omega^2 y \longrightarrow (4)$$

Since maximum value of displacement (y) is amplitude, hence

$$a_{max} = -\omega^2 A \longrightarrow (5)$$

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