1. Establish the relationship between AC and MC by the Quotient rule of differentiation.

We know when AC falls; AC > MC.

when AC increases; AC < MC.

and when AC is minimum; AC = MC.

Assume that total cost = C and Q = output

$\therefore$ $AC = \frac{C}{Q}$ and $MC = \frac{d(C)}{dQ}$.

when AC falls, the slope of AC is negative

i.e, $\frac{d(AC)}{dQ} < 0$.

$$\frac{d}{dQ}\left(\frac{C}{Q}\right) = \frac{Q\frac{dC}{dQ} - C\frac{dQ}{dQ}}{Q^2} < 0.$$

$$\Rightarrow \frac{Q\frac{dC}{dQ}}{Q^4} - \frac{C}{Q^2} < 0$$

$$\Rightarrow \frac{1}{Q}\left(\frac{dC}{dQ} - \frac{C}{Q}\right) < 0.$$

$$\Rightarrow \frac{dC}{dQ} - \frac{C}{Q} < 0.$$

$$\Rightarrow MC - AC < 0.$$

$$\Rightarrow MC < AC$$

$$\therefore AC > MC.$$

When AC increases, then $\frac{d}{dQ}\left(\frac{C}{Q}\right) > 0$.

$$\frac{d}{dQ}\left(\frac{C}{Q}\right) = \frac{Q\frac{dC}{dQ} - C\frac{dQ}{dQ}}{Q^2} > 0.$$

$$\Rightarrow \frac{Q\frac{dC}{dQ}}{Q^2} - \frac{C}{Q^2} > 0$$

$$\Rightarrow \frac{1}{Q}\left(\frac{dC}{dQ} - \frac{C}{Q}\right) > 0$$

$$\Rightarrow MC - AC > 0$$

$$\therefore MC > AC.$$

When AC is minimum, then $\frac{d}{dQ}\left(\frac{C}{Q}\right) = 0$.

$$\frac{d}{dQ}\left(\frac{C}{Q}\right) = \frac{Q\frac{dC}{dQ} - C\frac{dQ}{dQ}}{Q^2} = 0.$$

$$\Rightarrow \frac{Q\frac{dC}{dQ}}{Q^2} - \frac{C}{Q^2} = 0$$

$$\Rightarrow \frac{1}{Q}\left(\frac{dC}{dQ} - \frac{C}{Q}\right) = 0.$$

$$MC = AC.$$

$TC = 50 - 2Q + 7Q^2 + Q^3$ find MC at $Q = 5$, AFC, AVC. AC

As. $MC = 143$

$TC = \frac{1}{3}Q^3 + 6Q^2 + 12Q$ find AC & MC.

$TC = 2q^2 + 3q + 10$, find FC, AC and MC when $q = 10$.

Establish the relationship between AC and MC with the product rule of differentiation.

Assume that AC as a fun of quantity Produced, such that TC = AC × quantity. $AC = C(q)$.

$$= C(q) \times q.$$

$$MC = \frac{d}{dq}\left[C(q) \times q\right]$$

$$= C(q)\frac{dq}{dq} + q \cdot C'(q).$$

$$= C(q) + q \cdot C'(q)$$

$$MC = AC + q \cdot C'(q)$$

$$MC - AC = q \cdot C'(q)$$

Here $C'(q)$ represents the slope of the AC curve, when the slope of AC curve is downward, then $C'(q) < 0$.

$\therefore MC - AC < 0$. Since $q > 0$.

or, $MC < AC$.

$\therefore AC > MC$.

But when the slope of AC curve is upward, then $C'(q) > 0$.

$MC - AC > 0$.

$\therefore MC > AC$.

Again when AC curve is it's minimum point, then $C'(q) = 0$

$MC - AC = 0$.

or, $MC = AC$.