TWO-WAY CLASSIFICATION:-

Suppose we have two factors 'A' and 'B' where 'A' has p-levels and B has q-levels. Say $A_1, A_2 \cdots A_p$ and $B_1, B_2 \cdots B_q$. Let y_{ij} $(i=1,2,\cdots p;\ j=1,2,\cdots q)$ denotes the observations for the iTh level of 'A' and jTh level of 'B'.

The scheme of classification is given by

A \ B	B_1	B_2	B_j	B_q	Mean
A_1	y_{11}	y_{12}	y_{1j}	y_{1q}	$\bar{y}_{1o}$
A_2	y_{21}	y_{22}	y_{2j}	y_{2q}	$\bar{y}_{2o}$
⋮	⋮	⋮	⋮	⋮	⋮
A_i	y_{i1}	y_{i2}	y_{ij}	y_{iq}	$\bar{y}_{io}$
⋮	⋮	⋮	⋮	⋮	⋮
A_p	y_{p1}	y_{p2}	y_{pj}	y_{pq}	$\bar{y}_{po}$
Mean	$\bar{y}_{o1}$	$\bar{y}_{o2}$	$\bar{y}_{oj}$	$\bar{y}_{oq}$	$\bar{y}_{oo}$

Where, $\bar{y}_{io} = \frac{1}{q}\sum_{j=1}^{q} y_{ij}$; $\bar{y}_{oj} = \frac{1}{p}\sum_{i=1}^{p} y_{ij}$

and

$$\bar{y}_{oo} = \text{Grand Mean} = \frac{1}{pq}\sum_{i=1}^{p}\sum_{j=1}^{q} y_{ij}.$$

Here The fixed effect model is

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \longrightarrow (1)$$

P.T.O

μ = General Mean effect

α_i = Effect due to ith level of factor 'A'

β_j = Effect due to jth level of factor 'B'.

ϵ_{ij} = Error effect

Here we made the following assumptions:-

(i) ϵ_{ij} are independent and identically (iid) $N(0, \sigma^2)$

(ii) $\sum_i \alpha_i = 0 = \sum_j \beta_j$

The least square estimates of μ, α_i and β_j are obtained by minimising the sum of square of errors i.e by minimising

$$S = \sum_{i=1}^{p} \sum_{j=1}^{q} \epsilon_{ij}^2 = \sum_{i=1}^{p} \sum_{j=1}^{q} (y_{ij} - \mu - \alpha_i - \beta_j)^2$$

Now, $\frac{\partial S}{\partial \mu} = 0$

$$\Rightarrow -2 \sum_i \sum_j (y_{ij} - \mu - \alpha_i - \beta_j) = 0$$

$$\Rightarrow \sum_i \sum_j y_{ij} - pq\mu - q \sum_{i=1}^{p} \alpha_i - p \sum_{j=1}^{q} \beta_j = 0$$

$$\Rightarrow pq\mu = \sum_i \sum_j y_{ij} \qquad \left[\because \sum_i \alpha_i = 0 \sum_j \beta_j\right]$$

$\therefore \mu = \bar{y}_{00}$ $\qquad \therefore \hat{\mu} = \bar{y}_{00}$

Again $\frac{\partial S}{\partial \alpha_i} = 0$

$$\Rightarrow -2 \sum_{j=1}^{q} (y_{ij} - \mu - \alpha_i - \beta_j) = 0$$

$$\Rightarrow \sum_{j=1}^{q} y_{ij} - q\mu - q\alpha_i - \sum_{j=1}^{q} \beta_j = 0 \quad \left[\because \sum_j \beta_j = 0\right.$$

$$\Rightarrow q\alpha_i = \sum_{j=1}^{q} y_{ij} - q\mu.$$

$$\Rightarrow \alpha_i = \frac{1}{q}\sum_{j=1}^{q} y_{ij} - \mu. \Rightarrow \alpha_i = \bar{y}_{i0} - \bar{y}_{00}.$$

$$\therefore \hat{\alpha}_i = \bar{y}_{i0} - \bar{y}_{00}.$$

Similarly,

$$\beta_j = \bar{y}_{0j} - \bar{y}_{00}.$$

Now putting the values of μ, α_i & β_j in eq^n no (1), we get

$$y_{ij} = \bar{y}_{00} + (\bar{y}_{i0} - \bar{y}_{00}) + (\bar{y}_{0j} - \bar{y}_{00}) + \cancel{(y_{ij} + \bar{y}_{00} - \bar{y}_{i0} - \bar{y}_{0j}} + \epsilon_{ij}$$

$$y_{ij} = \cancel{\bar{y}_{00}} + \bar{y}_{i0} - \cancel{\bar{y}_{00}} + \bar{y}_{0j} - \bar{y}_{00} + \epsilon_{ij}$$

$$\Rightarrow y_{ij} + \bar{y}_{00} - \bar{y}_{i0} - \bar{y}_{0j} = \epsilon_{ij}$$

$$\Rightarrow \epsilon_{ij} = y_{ij} + \bar{y}_{00} - \bar{y}_{i0} - \bar{y}_{0j}$$

~~Putting both the sides and summing over i and j we get~~

Putting the values of μ, α_i, β_j and ϵ_{ij} in equation no. (1), we

$$\sum_i \cancel{\sum_j (y_{ij} - \bar{y}_{00})}$$

$$y_{ij} = \bar{y}_{00} + (\bar{y}_{i0} - \bar{y}_{00}) + (\bar{y}_{0j} - \bar{y}_{00}) + (y_{ij} + \bar{y}_{00} - \bar{y}_{i0} - \bar{y}$$

$$\Rightarrow (y_{ij} - \bar{y}_{00}) = (\bar{y}_{i0} - \bar{y}_{00}) + (\bar{y}_{0j} - \bar{y}_{00}) + (y_{ij} + \bar{y}_{00} - \bar{y}_{i0}$$

Now squaring both sides and summing over i and j we get.