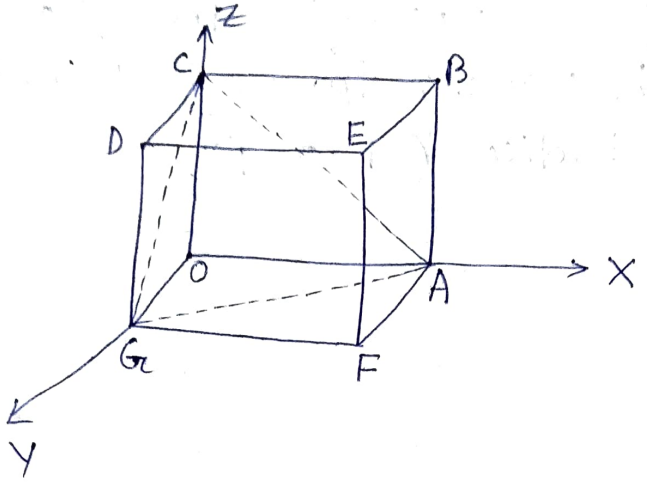
*) Considering the case of a cubic as shown in the fig. below, we shall consider the three kinds of planes which differ in their positions

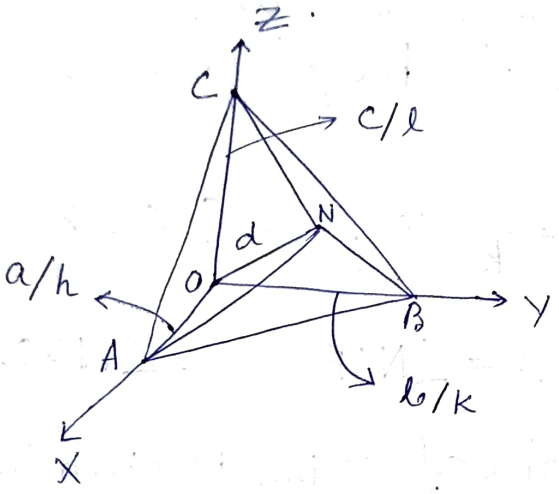


The face EBAF cuts the X-axis at a point A and is parallel to the two other axes i.e, y and z. When a face is parallel to an axis, its intercept on that axis is infinite. Suppose that the sides of the cubic is one unit in length then the intercepts made by this plane on the three axes are in the ratio $1:\infty:\infty$. As the Miller indices are reciprocals of these intercepts, the indices are $\frac{1}{1}:\frac{1}{\infty}:\frac{1}{\infty}$ i.e, 1:0:0. This face is called cubic face and is expressed as (100) face.

The face CBFG cuts the OY and OZ axes and is parallel to OX axis. The intercepts will be in the ratio $\infty:1:1$ and the Miller indices will be (011). This face is diagonal face and is expressed as (011) face.

The face CAG cuts equal intercepts on the three axes. The indices are (1 1 1). The face is called cubic diagonal face and is expressed as (1 1 1) face.

Interplaner distance (spacing) of lattice plane



We shall derive a formula for the spacing between two parallel plane in a given cell. For convenience, we shall take a simple unit cell in which co-ordinate axes are orthogonal (axes are mutually perpendicular) i.e; cubic tetragonal and orthorhombic cells. We can then use Cartesian co-ordinate system for calculating interplaner spacing. As shown in the figure OX, OY and OZ are orthogonal axes. The origin O is taken at any lattice point. We consider any set of crystal planes defined by the Miller indices (h k l). Suppose the reference plane passing through the origin and the next plane cuts the intercepts a/h, b/k and c/l on X, Y and Z axes respectively.

A normal ON is drawn to the plane ABC from the origin. The length d of this normal from the origin to the plane will be the distance between adjacent planes which is called interplaner distance.

We have to find out the expression for d in terms of a, b and c and h, k, l. Since d is normal to the plane ABC we can write

$$d = \frac{a}{h}\cos\alpha = \frac{b}{k}\cos\beta = \frac{c}{l}\cos\gamma$$

where $\alpha = \angle NOA$, $\beta = \angle NOB$, $\gamma = \angle NOC$.

The law of direction of cosine is given by

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

or, $$d^2\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right] = 1$$

or, $$d^2 = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-1}$$

or, $$d = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-1/2} \longrightarrow (1)$$

This expression is the interplaner distance. The relation is applicable to the primitive lattice in cubic orthorhombic and tetragonal system.

For tetragonal system $a = b$ so that

$$d = \left[\frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2} \right]^{-1/2}$$

For cubic system $a = b = c$ we get

$$d = \left[h^2 + k^2 + l^2 \right]^{-1/2} a^{-2x-1/2}$$

$$= a \left[h^2 + k^2 + l^2 \right]^{-1/2}$$

For ~~Miller~~ example for the Miller indices (100) the interplaner distance

$$d = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right]^{-1/2}$$

$$= \left[\frac{1}{a^2} \right]^{-1/2} = a$$

$\therefore d_{100} = a$.

For Miller indices (111) the interplaner distance

$$d = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right]^{-1/2}$$

$$= \left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \right]^{-1/2}$$

$$= \left[\frac{3}{a^2} \right]^{-1/2}$$

For cubic cell $a = b = c$

$$= 3^{-1/2} \; a^{-2x-1/2}$$

$$= 3^{-1/2}\, a$$

$$= a\, 3^{-1/2}$$

$$= \frac{a}{\sqrt{3}}$$

$$\therefore d_{111} = \frac{a}{\sqrt{3}}$$

For the Miller indices (1 1 0) the interplaner distance

$$d = \left[\frac{h^2}{a^2} + \frac{K^2}{b^2} + \frac{l^2}{c^2}\right]^{-1/2}$$

$$= \left[\frac{1}{a^2} + \frac{1}{b^2}\right]^{-1/2}$$

For cubic cell $a = b = c$

$$= \left[\frac{2}{a^2}\right]^{-1/2}$$

$$= 2^{-1/2}\, a^{2 \times -1/2}$$

$$= a\, \frac{1}{\sqrt{2}}$$

$$= \frac{a}{\sqrt{2}}$$

$$\therefore d_{110} = \frac{a}{\sqrt{2}}$$

For the Miller indices (0 1 0) the interplaner distance

$$d = \left[\frac{h^2}{a^2} + \frac{K^2}{b^2} + \frac{l^2}{c^2}\right]^{-1/2}$$

$$= \left[\frac{1}{b^2}\right]^{-1/2}$$

$$= b^{-2 \times -1/2}$$

$$= b$$

$$\therefore d_{010} = b$$

For the Miller indices (1 2 1) the interplaner distance

$$d = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-1/2}$$

$$= \left[\frac{1}{a^2} + \frac{4}{b^2} + \frac{1}{c^2}\right]^{-1/2}$$ | For cubic cell $a = b = c$

$$= \left[\frac{1+4+1}{a^2}\right]^{-1/2}$$

$$= \left[\frac{6}{a^2}\right]^{-1/2}$$

$$= 6^{-1/2} \times a^{-2 \times -1/2}$$

$$= \frac{a}{\sqrt{6}} \quad \therefore d_{121} = \frac{a}{\sqrt{6}}$$

For the Miller indices (2 1 1) the interplaner distance

$$d = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-1/2}$$

$$= \left[\frac{4}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right]^{-1/2}$$

$$= \left[\frac{4+1+1}{a^2}\right]^{-1/2} \quad \text{For cubic cell } a=b=c$$

$$= 6^{-1/2}\ a^{-2\times -1/2}$$

$$= \frac{a}{\sqrt{6}}$$

$$\therefore d_{211} = \frac{a}{\sqrt{6}} \quad //$$

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Angle between Two Planes having Miller indices $(h_1 k_1 l_1)$ and $(h_2 k_2 l_2)$

We have the unit vector normal to the plane $(h_1 k_1 l_1)$.

Then

$$\hat{n}_1 = \frac{\hat{i}\frac{h_1}{a} + \hat{j}\frac{k_1}{b} + \hat{k}\frac{l_1}{c}}{\sqrt{\left(\frac{h_1}{a}\right)^2 + \left(\frac{k_1}{b}\right)^2 + \left(\frac{l_1}{c}\right)^2}}$$

For a cubic system, $a=b=c$,

$$\therefore \hat{n}_1 = \frac{(\hat{i}h_1 + \hat{j}k_1 + \hat{k}l_1)}{\sqrt{h_1^2 + k_1^2 + l_1^2}}$$

Similarly, the unit vector normal to the plane (h_2, k_2, l_2) of a cubic system is

$$\hat{n}_2 = \frac{(\hat{i}h_2 + \hat{j}k_2 + \hat{k}l_2)}{\sqrt{h_2^2 + k_2^2 + l_2^2}}$$

Now

$$\hat{n}_1 \cdot \hat{n}_2 = |\hat{n}_1||\hat{n}_2| \cos\theta \quad [\text{where } \theta \text{ is the angle between the planes}]$$

$$= \frac{\hat{i}h_1 + \hat{j}k_1 + \hat{k}l_1}{\sqrt{h_1^2 + k_1^2 + l_1^2}} \cdot \frac{\hat{i}h_2 + \hat{j}k_2 + \hat{k}l_2}{\sqrt{h_2^2 + k_2^2 + l_2^2}}$$

$$= \frac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$$

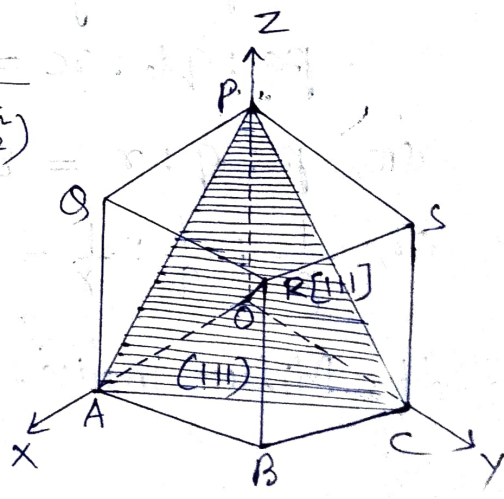
$$\therefore \theta = \cos^{-1}\left[\frac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}\right]$$

Q.1 Find the angle between the plane (111) and plane direction [111].

Ans:- The (111)-plane in a cubic unit cell is represented by PAC. PA is on the (111)-plane and the direction of PA is represented by $[10\bar{1}]$. The angle between the direction $[10\bar{1}]$ and plane [111] is the angle between the plane (111) and the direction [111].

If θ be the angle between (111) and [111], then

$$\theta = \cos^{-1}\frac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$$



Here $h_1 = 1$, $h_2 = 1$
$k_1 = 0$ and $k_2 = 1$
$l_1 = -1$, $l_2 = 1$

$$\therefore \theta = \cos^{-1}\frac{1\times1 + 1\times0 + (-1\times1)}{\sqrt{1^2 + 1^2 + (-1)^2}\sqrt{(1)^2 + (0)^2 + (1)^2}}$$

$$= 0 \quad //$$

Q In an orthorhombic crystal a lattice plane cuts intercepts of lengths $3a$, $-2b$ and $3c/2$ along three axes. Deduce the Miller indices of the plane, where $\vec{a}, \vec{b}, \vec{c}$ are vectors of the unit cell.

Sol^{n}:- If the intercepts of the plane in the three crystal axes are pa, qb and rc respectively, then

$$pa : qb : rc = 3a : (-2b) : \frac{3c}{2}$$

or, $$p : q : r = 3 : (-2) : \frac{3}{2}$$

or, $$\frac{1}{p} : \frac{1}{q} : \frac{1}{r} = \frac{1}{3} : \left(-\frac{1}{2}\right) : \frac{2}{3} = 2 : (-3) : 4.$$

$\therefore$ Miller indices of the plane are $(2\bar{3}4)$.

Q. Find the Miller indices of a plane having intercepts of $8a$, $4b$ and $2c$ on the a-, b- and c-axes respectively.

Ans:- If the intercepts of the plane on a-, b- and c-axes respectively be pa, qb and rc, then

$$pa : qb : rc = 8a : 4b : 2c$$

or, $$p : q : r = 8 : 4 : 2$$

or, $$\frac{1}{p} : \frac{1}{q} : \frac{1}{r} = \frac{1}{8} : \frac{1}{4} : \frac{1}{2} = 1 : 2 : 4$$

$\therefore$ the Miller indices of the plane are (124).

Q. In an orthorhombic crystal a plane makes intercepts 2.93 mm, 4.47 mm and 2.35 mm along the three crystallographic axes, the corresponding primitives being 3.05 Å, 6.99 Å and 4.90 Å. Deduce the Miller indices of the cleavage plane.

Sol:- We have

$$p \times 3.05 : q \times 6.99 : r \times 4.90 = 2.93 : 4.47 : 2.35$$

$$\text{or, } p : q : r = \frac{2.93}{3.05} : \frac{4.47}{6.99} : \frac{2.35}{4.90}$$

$$= 0.96 : 0.64 : 0.48$$

$$= 6 : 4 : 3$$

$$\therefore \frac{1}{p} : \frac{1}{q} : \frac{1}{r} = \frac{1}{6} : \frac{1}{4} : \frac{1}{3} = 2 : 3 : 4$$

Hence, Miller indices of the plane are (234).

Q. Draw the following planes and directions in the case of an FCC structure (i) (100) (ii) (110) (iii) (112).

Sol:- (i) For (100)-plane, $h=1$, $k=0$, $l=0$

The reciprocals are $\frac{1}{1}, \frac{1}{0}, \frac{1}{0}$, i.e. $1, \infty, \infty$.

The plane is shown in fig (1)

The direction [100] is given by the normal to (100)-plane and passing through the origin, i.e; SR.

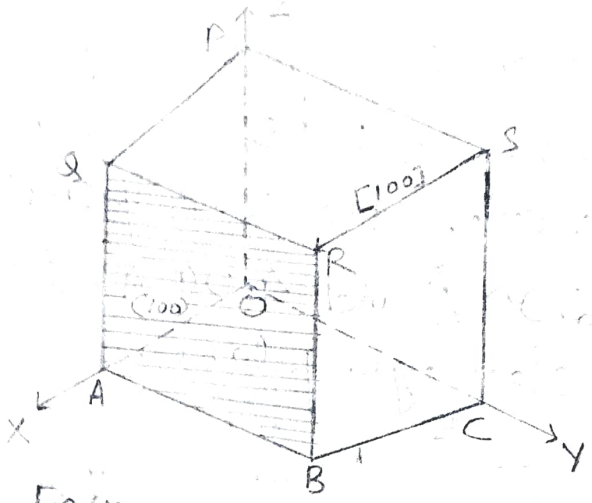


Fig.(1) (100) - plane.

(ii) For (110) - plane, $h=1, k=1, l=0$

$\therefore \frac{1}{h}=1, \frac{1}{k}=1, \frac{1}{l}=\infty$

The plane is shown in fig.(2).

The direction [110] is the perpendicular to (110) - plane and passing through O, i.e., OB.

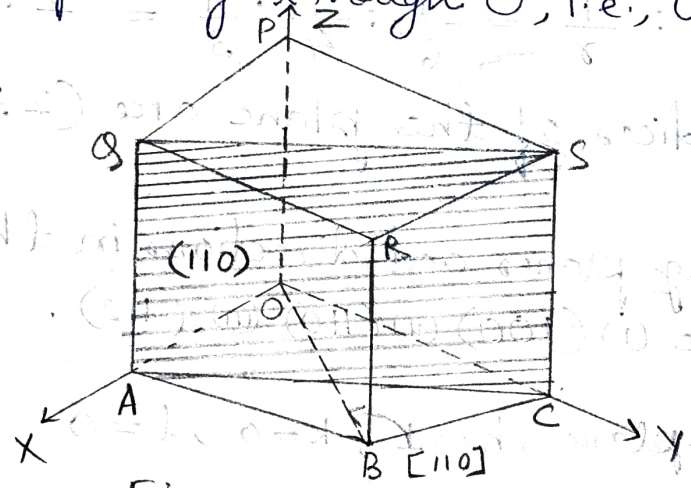


Fig.2 (110) - plane

(iii) For (112) - plane, $h=1, k=1, l=2$

$\therefore \frac{1}{h}=1, \frac{1}{k}=1, \frac{1}{l}=0.5.$

The plane is shown in fig. 3.

The direction [112] is the perpendicular to (112) - plane and passing through O, i.e., OR.

(iii) (022)-Plane : Here

$$h = 0, \quad \therefore \frac{1}{h} = \frac{1}{0} = \infty$$

$$k = 2, \quad \therefore \frac{1}{k} = \frac{1}{2} = 0.5$$

and $l = 2, \quad \therefore \frac{1}{l} = \frac{1}{2} = 0.5.$

So, the (022)-plane has intercepts ∞, 0.5 and 0.5 along the X-, Y- and Z-directions respectively. [Fig. 1 (c)].

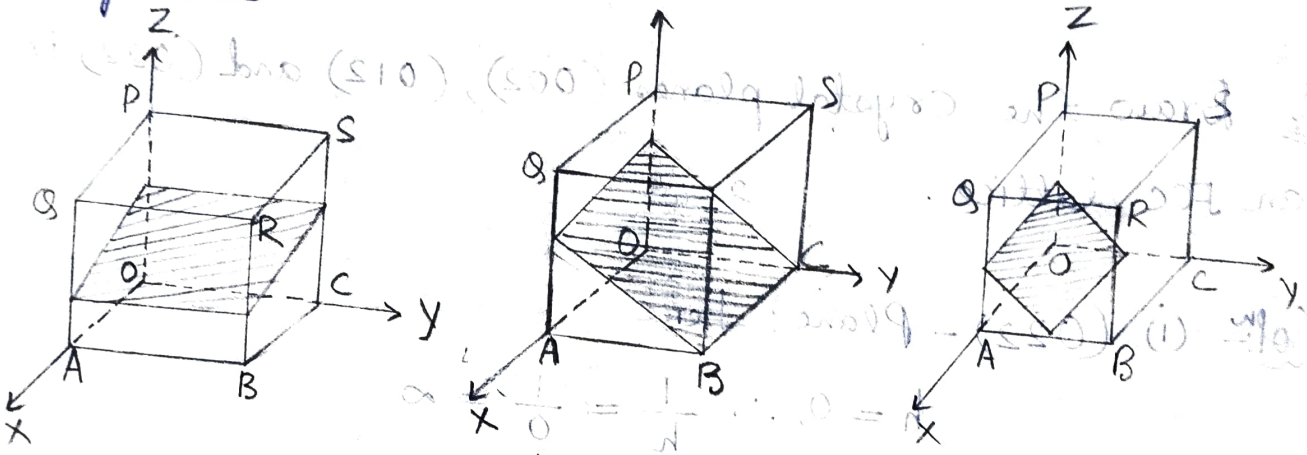


Fig. 1 (a) (022)-plane — Fig. 1 (b) (012)-Plane — Fig. 1 (c) (022) Plane