Exercise 3.3

Question 1:

Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Answer

(i) Let $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Question 2:

If , $A=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}$ then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

Answer We have:

$$A'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}, B'=\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}$$

(i)

$$A+B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}=\begin{bmatrix}-5 & 3 & -2\\ 6 & 9 & 9\\ -1 & 4 & 2\end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}$$

$$A'+B'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}$$

Hence, we have verified that $(A+B)'=A'+B'$

(ii)

$$A-B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}=\begin{bmatrix}3 & 1 & 8\\ 4 & 5 & 9\\ -3 & -2 & 0\end{bmatrix}$$

$$\therefore (A-B)'=\begin{bmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{bmatrix}$$

$$A'-B'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 4 & -3\\ 1 & 5 & -2\\ 8 & 9 & 0\end{bmatrix}$$

Hence, we have verified that $(A-B)'=A'-B'$.

Question 3:

If , $A'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$ then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

Answer

(i) It is known that $A=(A')'$

Therefore, we have:

$$A=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}$$

$$B'=\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$$

$$A+B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$$

$$A'+B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$$

Thus, we have verified that $(A+B)' = A'+B'$.

(ii)

$$A-B=\begin{bmatrix}3 & -1 & 0\\4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\1 & 2 & 3\end{bmatrix}=\begin{bmatrix}4 & -3 & -1\\3 & 0 & -2\end{bmatrix}$$

$$\therefore (A-B)'=\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$$

$$A'-B'=\begin{bmatrix}3 & 4\\-1 & 2\\0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\2 & 2\\1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$$

Thus, we have verified that $(A-B)'=A'-B'$.

Question 4:

If $A'=\begin{bmatrix}-2 & 3\\1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$, then find $(A+2B)'$

Answer

We know that $A=(A')'$

$$\therefore A=\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}$$

$$\therefore A+2B=\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}+2\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}+\begin{bmatrix}-2 & 0\\2 & 4\end{bmatrix}=\begin{bmatrix}-4 & 1\\5 & 6\end{bmatrix}$$

$$\therefore (A+2B)'=\begin{bmatrix}-4 & 5\\1 & 6\end{bmatrix}$$

Question 5:

For the matrices *A* and *B*, verify that $(AB)' = B'A'$ where

(i) $A=\begin{bmatrix}1\\-4\\3\end{bmatrix}, B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

(ii) $A=\begin{bmatrix}0\\1\\2\end{bmatrix}, B=\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

Answer

(i)

$$AB=\begin{bmatrix}1\\-4\\3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix}=\begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}$$

$$\therefore (AB)'=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$$

$$\text{Now, } A'=\begin{bmatrix}1 & -4 & 3\end{bmatrix}, B'=\begin{bmatrix}-1\\2\\1\end{bmatrix}$$

$$\therefore B'A'=\begin{bmatrix}-1\\2\\1\end{bmatrix}\begin{bmatrix}1 & -4 & 3\end{bmatrix}=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$$

Hence, we have verified that $(AB)'=B'A'$.

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

Question 6:

If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

Answer

(i)

$$A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha)\\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

(ii)

$$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$
$$= \begin{bmatrix} (\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha) \\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha) \end{bmatrix}$$
$$= \begin{bmatrix} \sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, we have verified that $A'A = I$.

Question 7:

(i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix

Answer

(i) We have:

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$\therefore A' = A$

Hence, A is a symmetric matrix.

(ii) We have:

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$\therefore A' = -A$

Hence, A is a skew-symmetric matrix.

Question 8:

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Answer

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

(i) $$A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

Hence, $(A + A')$ is a symmetric matrix.

(ii) $$A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$$

Hence, $(A - A')$ is a skew-symmetric matrix.

Question 9:

Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A=\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Answer

The given matrix is $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

$$A+A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}+\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\text{Now, } A-A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}-\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A')=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

Question 10:

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Answer

(i)

$$\text{Let } A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}, \text{then } A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\text{Now, } A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now, } P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing *A* as the sum of *P* and *Q*:

$P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$

(ii)

Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, then $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P+Q=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}=A$$

(iii)

$$\text{Let } A=\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}, \text{ then } A'=\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}+\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}=\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}-\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}=\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ -\frac{5}{2} & 0 & 3\\ -\frac{3}{2} & -3 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & -\frac{5}{2} & -\frac{3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P+Q=\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}+\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}=A$$

(iv)

$$\text{Let } A=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}, \text{ then } A'=\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\text{Now } A+A'=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}-\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing *A* as the sum of *P* and *Q*:

$$P+Q=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}+\begin{bmatrix}0 & 3\\ -3 & 0\end{bmatrix}=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}=A$$

Question 11:

If *A*, *B* are symmetric matrices of same order, then *AB* − *BA* is a

A. Skew symmetric matrix **B.** Symmetric matrix

C. Zero matrix **D.** Identity matrix

Answer

The correct answer is A.

A and *B* are symmetric matrices, therefore, we have:

$$A' = A \text{ and } B' = B \qquad ...(1)$$

$$\begin{aligned}\text{Consider } (AB-BA)' &= (AB)'-(BA)' && \left[(A-B)' = A'-B'\right]\\ &= B'A'-A'B' && \left[(AB)' = B'A'\right]\\ &= BA-AB && [\text{by } (1)]\\ &= -(AB-BA)\end{aligned}$$

$$\therefore (AB-BA)' = -(AB-BA)$$

Thus, (*AB* − *BA*) is a skew-symmetric matrix.

Question 12:

If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then $A+A'=I$, if the value of α is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. п

D. $\frac{3\pi}{2}$

Answer

The correct answer is B.

$$A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$\Rightarrow A'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\text{Now, } A+A'=I$$

$$\therefore \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$2\cos\alpha=1$$

$$\Rightarrow \cos\alpha=\frac{1\pi}{2}=\cos\frac{}{3}$$

$$\therefore \alpha=\frac{\pi}{3}$$